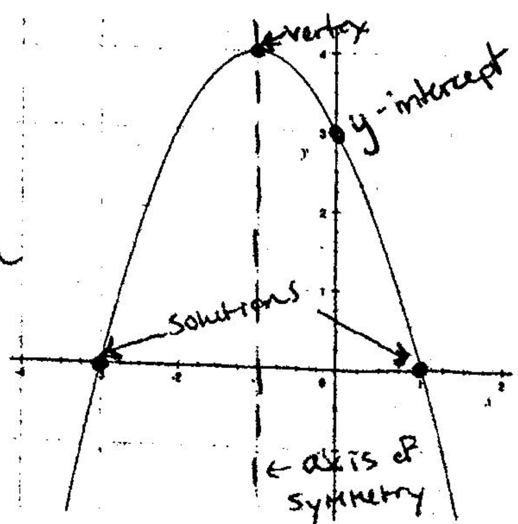
REVIEW – QUADRATIC FUNCTIONS KEY

BASIC CONCEPT QUESTIONS

1. Name and write the equation of the two forms we use to represent quadratic functions.
   Standard Form: $y=ax^2+bx+c$ Vertex Form: $y=a(x-h)^2+k$
2. Circle all of the following functions that are quadratic functions.
   $y=-\frac{1}{2}x+3$ $y=8x^2-7$ $y=x^2$ $y=9x^3-5x^2+2$ $y=-6(x+1)^2$
3. Given an equation, how do you know if a parabola is concave up or concave down?
   If a is positive → concave up a negative → concave down
4. Given an equation, how do you know if the parabola will be skinnier or wider than the parent function? $|a|>1$ → skinnier $|a|<1$ → wider * negatives don't matter! Use absolute value!
5. What do *c* and *k* change about the graph?
   Shift up + down (vertical translation)
6. What does the *h* value change about the graph?
   shift left and right (horizontal translation) * opposite sign as what you see in ( ) left + right −
7. What is the vertex?
   Max or Min value, turning point. Always on axis of symmetry.
8. How do you find the vertex of a quadratic function
   a. From Standard Form? $x=\frac{-b}{2a}$ + plug back in
   b. From Vertex Form? (h, k)
9. From the equation, how can you tell if the vertex will be a maximum or a minimum?
   $a>0$ → Min $a<0$ → Max
10. What is the axis of symmetry? $x=$
    Line through vertex down middle of parabola (that cuts it into 2 symmetric halves)
11. How do you find the axis of symmetry? - x value of vertex
    a. From Standard Form? $x=-\frac{b}{2a}$
    b. From Vertex Form? $x=h$
12. How do you find the y-intercept of any function?
    plug in $x=0$
13. List all the other names for "solutions" of a quadratic function.
    roots, zeros, x-intercepts
14. What do you need to be sure is true about your function before you start finding solutions?
    that it $=0$
15. What are the 3 methods for finding solutions of a quadratic function?
    1 Graphing (+ find x-intercepts) 2 Factoring + Zero product property 3 Quadratic Formula
16. State the quadratic formula. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
17. What is the discriminant? b^2-4ac
18. What does the value of the discriminant tell you about your function if it is
    a. positive 2 real solutions
    b. negative 2 complex solutions
    c. zero 1 real solution
19. Label all of the key features (vertex, axis of symmetry, solutions, y-intercept) on the graph to the right.



20. Give either definition for *i*.
    $i=\sqrt{-1}$ $i^2=-1$

PRACTICE PROBLEMS

21) $(3+7i)-(9+4i) = 3+7i-9-4i = \boxed{-6+3i}$

22) $(3+7i)(9+4i) = 27+12i+63i+28i^2$

$= 27+75i-28 = \boxed{-1+75i}$

23) $\sqrt{-36} = \boxed{6i}$

24) $f(3i) = (3i)^2 = 9i^2 = 9(-1) = \boxed{-9}$

25) $\sqrt{x^2} = \sqrt{-81}$ $\boxed{x = \pm 9i}$

26) $(\sqrt{x+4})^2 = (2i)^2$

$x+4 = 4i^2$

$x+4 = -4$

$x = -8$

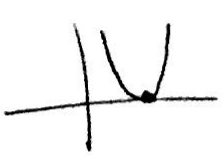
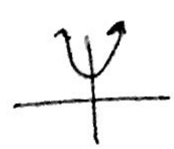
27) $\dfrac{2-3i}{6+5i} \cdot \dfrac{6-5i}{6-5i} = \dfrac{12-10i-18i+15i^2}{36-30i+30i-25i^2} = \dfrac{12-28i-15}{36+25} = \dfrac{-3-28i}{61}$

or $\dfrac{-3}{61} - \dfrac{28i}{61}$

28) A. $f(0) = 4(0)^2-24(0)-7 = -7$ $\boxed{(0,-7)}$

B. $x = \dfrac{-b}{2a} = \dfrac{-(-24)}{2(4)} = \dfrac{24}{8} = 3$ $x=3$

C. $f(3) = 4(3)^2-24(3)-7 = -43$ $\boxed{(3,-43)}$ Min (since $a>0$)

29) A.  B.  C. 

30) $\dfrac{6x^2}{6} = \dfrac{150}{6}$ $x^2 = 25$ $\boxed{x = \pm 5}$

31) $x = 1, 4$ or $(1,0)$; $(4,0)$

32) $9x^2 = 4x$ $-4x \quad -4x$ $9x^2 - 4x = 0$ $x(9x-4) = 0$ $x = 0$ or $9x - 4 = 0$ $x = 4/9$

33) $x^2 - 6x - 7 = (x-7)(x+1) = 0$ $x = 7, -1$

34) $x = \dfrac{-6 \pm \sqrt{76}}{2(-2)} = \dfrac{-6 \pm 2\sqrt{19}}{-4} = \boxed{\dfrac{-3 \pm \sqrt{19}}{-2}}$ (or $x \approx 3.68, -0.68$)

$\sqrt{76} = \sqrt{4}\sqrt{19} = 2\sqrt{19}$

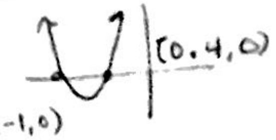
35) $x = \dfrac{-2 \pm \sqrt{-80}}{6} = \dfrac{-2 \pm 4i\sqrt{5}}{6} = \boxed{\dfrac{-1 \pm 2i\sqrt{5}}{3}}$

$\sqrt{80} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$

36) $x^2 + 7x + 10 = 0$

$(x+5)(x+2) = 0$ $x = -5, -2$

37) Graph



Quad. Form.

$x = \dfrac{-7 \pm \sqrt{9}}{2(5)} = \dfrac{-7 \pm 3}{10} \nearrow -\frac{2}{5}$, $\searrow -1$

Factor

$(5x+2)(x+1) = 0$

$5x + 2 = 0 \qquad x + 1 = 0$

$x = -2/5, -1$

38) $\frac{b}{2} = \frac{6}{2} = 3 \qquad (\frac{b}{2})^2 = 3^2 = 9$

$x^2 + 6x = 3$
$+9 \qquad +9$

$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$x^2 + 6x + 9 = 12$

$(x+3)^2 = 12$

$x + 3 = \pm\sqrt{12} = \pm 2\sqrt{3}$
$-3 \qquad\qquad -3$

$x = -3 \pm 2\sqrt{3}$

39) Complete the □ $x^2 - \frac{s}{2} = 4r$ $\quad \frac{b}{2} = \frac{1}{2}\cdot\frac{-s}{2} = \frac{-s}{4}$, $\left(\frac{b}{2}\right)^2 = \left(\frac{-s}{4}\right)^2 = \frac{s^2}{16}$

$$+\frac{s^2}{16} \quad + \frac{s^2}{16}$$

Remember it's always $\left(x+\frac{b}{2}\right)^2$

$$x^2 - \frac{s}{2} + \frac{s^2}{16} = 4r + \frac{s^2}{16}$$

$$\left(x - \frac{s}{4}\right)^2 = \frac{64+s^2}{16}$$

$$x - \frac{s}{4} = \pm\sqrt{\frac{64+s^2}{16}} = \frac{\pm\sqrt{64+s^2}}{4}$$

$$+\frac{s}{4} \qquad +\frac{s}{4}$$

$$x = \frac{s}{4} \pm \frac{\sqrt{64+s^2}}{4} = \boxed{\frac{s \pm \sqrt{64+s^2}}{4}}$$

40) Find the solutions, then add them together

$$x = \frac{-4 \pm \sqrt{320}}{2(2)} = \frac{-4 \pm 8\sqrt{5}}{4} = -1 \pm 2\sqrt{5}$$

$$\sqrt{320} = \sqrt{64}\sqrt{5} = 8\sqrt{5}$$

sum $\underbrace{-1+2\sqrt{5}}_{\text{solution 1}} + \underbrace{-1-2\sqrt{5}}_{\text{solution 2}} = \boxed{-2}$

41) $(ax+5)(bx+3) = abx^2 + 3ax + 5bx + 15 = 6x^2 + cx + 15$

$ab = 6 \qquad 3a + 5b = c$

and $a+b=5$ (given)

$\Downarrow$

$a=3$ and $b=2$

or $a=2$ and $b=3$

$\Rightarrow$ $3a+5b = 3(3)+5(2) = 19$

or $3a+5b = 3(2)+5(3) = 21$

$\boxed{19 \text{ or } 21}$

42) $y = a(x+3)(x-5)$ ⇒

$y = a(x^2-2x-15) = ax^2-2ax-15a$

$x = \frac{-b}{2a} = \frac{2a}{2a} = 1 = c$ $f(1) = a(1)^2 - 2a(1) - 15a$

$= a - 2a - 15a = \boxed{-16a = d}$

43) $y = \ (x-2)^2+12$

any $a > 1$

44) $f(x) = a(x-5)^2 - 6$ y-int $= 10 \Rightarrow (\overset{x}{0}, \overset{y}{10})$. Plug these in!

$10 = a(0-5)^2 - 6$

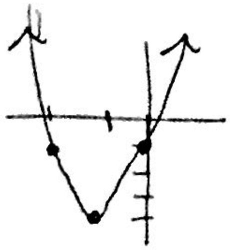
$10 = 25a - 6$

$16 = 25a$

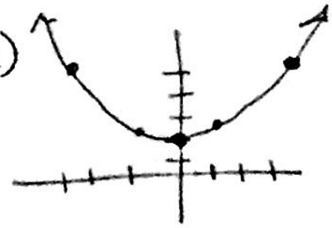
$a = \frac{16}{25}$

$\boxed{f(x) = \frac{16}{25}(x-5)^2 - 6}$

45) vertex $(-1,-4)$
$a = 3$



46) vertex $(0,2)$
$a = \frac{1}{3}$



47) vertex $= (3,0)$ ⇒ $f(x) = a(x-3)^2 + 0$

Now plug in any other point. (OR know that Δy for $\Delta x = 1$ on each side of the vertex is 1 so $a = 1$)

Ex: $(\overset{x}{2}, \overset{y}{1})$

$1 = a(2-3)^2 + 0$

$1 = a(-1)^2 + 0$

$1 = a$

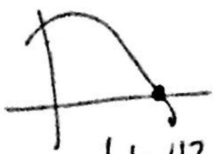
→ $\boxed{y = (x-3)^2}$

48) A. $t = \frac{-b}{2a} = \frac{-56}{2(-16)} = \frac{-56}{-32} = \boxed{1.75 \text{ seconds}}$

B. $h(1.75) = -16(1.75)^2 + 56(1.75) + 300 = \boxed{349 \text{ ft}}$

C. $t = 1.2$ $\quad f(1.2) = -16(1.2)^2 + 56(1.2) + 300 = \boxed{344.16 \text{ ft}}$

D. $0 = -16t^2 + 56t + 300$



$(6.42, 0)$

or $\quad x = \frac{-56 \pm \sqrt{22336}}{2(-16)} = \frac{-56 \pm 149.45}{-32}$

$\frac{93.45}{-32} = \cancel{-2.9}$ $\qquad \frac{-205.45}{-32} = \boxed{6.42 \text{ sec}}$

49) vertex $= (-1, 12)$ $\quad a = -3$ $\quad [9 = a(0+1)^2 + 12 \Rightarrow 9 = a + 12 \Rightarrow a = -3]$

A. $y = -3(x+1)^2 + 12$

B. roots are $x = 1$ and $x = -3$. Use the same a

$y = -3(x-1)(x+3)$

C. Distribute either A or B.

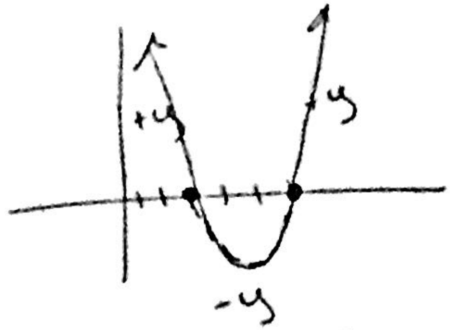
A. $y = -3(x+1)(x+1) + 12$
$= -3(x^2 + 2x + 1) + 12$
$= -3x^2 - 6x - 3 + 12$
$= -3x^2 - 6x + 9$

$-3(x-1)(x+3)$
$= -3(x^2 + 2x - 3)$
$= -3x^2 - 6x + 9$

50) $x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x = 6, 3$



When is y positive?

When is y negative?

⇓

$f(x) < 0$

From $x = 3$ to $x = 6$ $\boxed{(3, 6)}$ ← note: this is an interval NOT a point!

$\boxed{3 < x < 6}$

51) $P = 2x + 2l = 400$ $\quad A = lx$

$l = \dfrac{400 - 2x}{2} = 200 - x$ $\quad A = (200 - x)(x)$

$= 200x - x^2$ ← Maximize (find vertex)

$x = \dfrac{-200}{2(-1)} = \dfrac{-200}{-100} = 100$ $\quad l = 200 - x = 200 - 100 = 100$

$\boxed{100 \text{ ft} \times 100 \text{ ft}}$